The BAA Christmas Lecture

Black Holes: A road to unity for theoretical physics?

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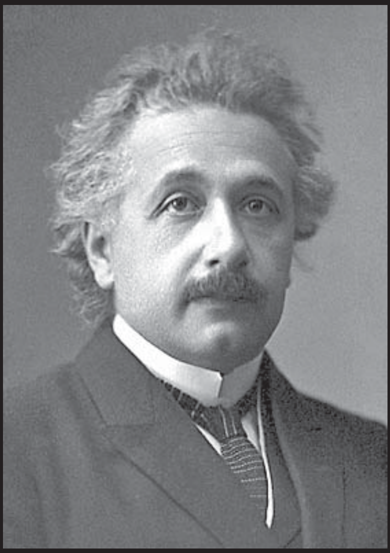

Presented here is a short paper based on the Christmas lecture given to the BAA on 2012 December 15 by the author , concerning black holes and how they might be used by theorists to unify quantum mechanics and general relativity. In Section 1 we examine the historical context of gravitation and relativity, then in Section 2 we see how these ideas were dramatically changed by Albert Einstein. In section 3 we look at how black holes arise naturally from relativity, and examine how they are thought to form in section 4. In section 5 we look at Hawking radiation while in section 6 we examine the implications of black holes for theoretical physics today.

Introduction

The black hole is perhaps one of the most exotic objects ever given form by the forces of nature. There was a time when such objects received little serious attention; the theoretical discovery of these objects was treated largely with indifference, many scientists at the time considering them to be yet another quirky aspect to the recently new theory of general relativity.

Today, black holes are treated very seriously. They are a much respected area of physics, and the last fifty years has seen some startling discoveries concerning these exotic objects – discoveries which have once more challenged the fundamental principles of the Universe which we had taken for granted. The continual study of black holes may even lead the way to that holy grail of modern physics, a theory of quantum gravity.

Before we can understand the importance of that however, we need to understand what a black hole is, how such an object can come to exist, and what effects the black hole might exhibit in a Universe like ours. Just as important is the historical development of the subject, for there was a time when powerful notions of absolute space and absolute time held sway, and to suggest otherwise was heresy. Einstein ushered in a brave new world, but this new world would take some time to find a friendly audience.

Let's go back to the start. To the time when it was thought the Universe operated like a vast orrery. Space was fixed, and time was the river that ran though it without deviation nor change in its eternal flow. An age when the perceived wisdom of Aristotle started to conflict with astronomical observation.

Galileo and Newton

Black holes are a natural consequence of Albert Einstein's theory of general relativity. This is our most successful theory of gravity; however it was not the first theory of gravity, nor indeed was Einstein *(above)* the first to develop a form of relativity – this idea that the laws of physics ought not be unique to the Earth.

Isaac Newton *(left)* is often credited as being the godfather of modern physics, but personally I think Galileo Galilei should be awarded this title. Galileo contributed a great deal to

Galileo Galilei (1636)

science, and in particular to astronomy. His observations of Jupiter provided the first unequivocal proof that satellites orbit planets other than the Earth, and thus further evidence for Copernicus' heliocentric model of the solar system, where the Sun, rather than the Earth, is at the centre of the system.

Galileo also experimented with gravity; he climbed the leaning tower of Pisa with various objects of different mass, and – no doubt to much amusement from the locals – dropped them from the tower. He noted that the objects all fell at the same rate, independent of their mass. These weren't just esoteric observations: they allowed Galileo to hypothesise that *the laws of physics are the same in any system moving at a constant speed in a straight line*. This, albeit in a rather more general form, would become one of the founding principles of special relativity some three hundred years in the future.

The man credited with the first formulation of gravity was Sir Isaac Newton. It is often said that Newton started thinking about gravity when he was at home and an apple dropped on his head – alas although the story is amusing it has no bearing in reality, although it is certainly true that Newton was at home when he started thinking about gravity, as Cambridge University had been closed due to the plague.

Newton was the first person to give a mathematical expression for gravity. This is essential; the language of science is mathematics. Mathematical models allow us to make predictions which can then be tested by observation to see how well a theory agrees with experiment. Newton's law of gravity states, quite simply, that *a point mass M attracts another point mass m with a force that is proportional to the product of their masses and inversely proportional to the square of the distance from their centres*. So:

$$F \propto \frac{Mm}{r^2}$$

We can remove the proportionality sign '∝' and replace it with an equal sign to give a full equation:

$$F = \frac{GMm}{r^2}$$

where G, the constant of proportionality, is called the universal gravitational constant.

Newton published his theory of gravity in his much celebrated work the *Principia* (1686) but there was some controversy. Robert Hooke claimed that Newton had obtained the inverse square law of gravity from him. It is true that in papers and lectures previous to 1686 Hooke had spoken about the role of gravity in celestial mechanics, however he had offered no mathematical formulations of these ideas, moreover his theory of gravity was not yet universal. While it can be agreed there there may have been some cross fertilisation between Hooke and Newton, it was Newton who introduced the first mathematical description of gravity and so, to my mind, is rightly credited with producing the first testable theory.

Newton's law of gravity was a major triumph; when combined with Kepler's laws of planetary motion it allowed the computation of planet and cometary orbits and provided a glimpse of the silent invisible mechanisms of the solar system. There were also a number of things it couldn't do. Newton's gravity could not account for Mercury's orbit, which undergoes a phenomenon known as perihelion precession. As a result of bring so close to the Sun, Mercury does not return exactly to the same point each time when it completes one orbit about the Sun. This makes the orbit of the planet precess (see Figure 1).

Perhaps the biggest critic of the theory was Newton himself. Newton was unhappy that his theory did not give any insight into what a gravitation field was; it simply provided a description of how it behaved between two bodies of different mass. He was also unhappy with the 'action at a distance' mechanism his theory apparently suggested. In spite of these shortcomings, Newtonian gravity held sway until the arrival of Einstein. Even today, Newtonian gravity is sufficient for getting spacecraft out to the planets.

Newton can't be blamed for the limitations of his theory; these were symptomatic of a much deeper problem in physics. Since the time of Aristotle two fundamental ideas had gone unchallenged: the notions of *absolute space* and *absolute time*. It seemed quite reasonable to suppose that space was a fixed background against which the events of the Universe moved, whilst time was a constant, running through every part of the cosmos at the same rate everywhere. If you abide by these principles, you can't get any further than Newtonian gravity. What was needed was no less than a revolution in the fundamental philosophy of space, time and movement.

The Einsteinian revolution

Albert Einstein was born on 1879 March 14. At school, he was not particularly gifted. He completed his PhD from the University of Zurich on 1905 April 30, but it was four other papers which he published that year that brought him to the attention of the academic world, in particular his paper on the special theory of relativity.

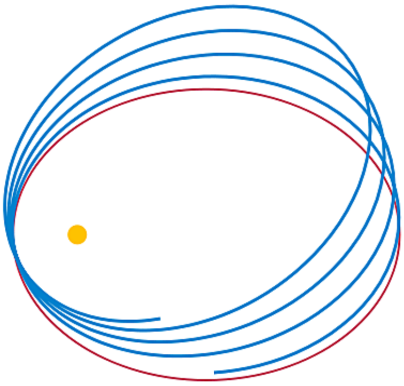

Figure 1. The perihelion precession of Mercury's orbit.

Einstein's theory of special relativity completely transformed modern physics; not only did it do away with the old notions of absolute space and time, it also introduced some new and rather startling fundamental concepts. At the heart of relativity are two postulates. These are:

(I): The laws of physics are the same in all frames of reference (a frame of reference is simply a coordinate system with a measure of length, height and width with its own clock). This is known as *the equivalence principle*.

(II): The speed of light in a vacuum is constant.

The result of these postulates means that everyone has their own concept of time (i.e. their own personal clock) called *proper time*. Moreover, a person's clock will tick slower, the faster they move. Time is not the same everywhere; it is measured differently depending on how fast you are moving. If you were on a spaceship approaching the speed of light, the ship's clock would get slower and slower. Einstein also discovered that as one moves faster towards the speed of light, the heavier and shorter one becomes (this is called Lorentz contraction). Indeed at the speed of light, anything with mass becomes infinitely heavy: the speed of light is the speed limit for the Universe. We also have special relativity to thank for that now famous equation $E=mc^2$ which tells us that matter and energy are equivalent.

Special relativity also comes with its own mathematical framework, and we have the Polish mathematician Hermann Minkowski to thank for this. Minkowski realized that the postulates central to relativity could be interpreted better if the structure of the Universe was thought of as a 4-dimensional framework he called *spacetime*. This geometric way of thinking about the Universe is, as we shall see, essential for understanding general relativity.

The spacetime of special relativity is a flat, 4-dimensional structure. Every point in it is called an *'event'* and has three spatial coordinates (up-down, left-right, backwards-forwards) and one time coordinate. Anyone moving through spacetime travels on their own *worldline* which has its own sense of proper time.

Since we have seen that nothing can travel faster than the speed of light, this means there are events in spacetime so far away from each other, they can never see each other. For example, there are galaxies in the Universe so far away their light can never reach us. This means that there is a 'horizon' around each point in spacetime. Anything beyond that horizon can never be observed.

Einstein had not only introduced new principles of space and time, he had generalised Galileo's result: all uniform motion is relative and there is no special frame of rest.

After the success of special relativity, Einstein turned his attention to gravity. Newtonian gravity was then the only serious theory of gravity, but the flaws in theory had pushed scientists to consider if there was a better theory waiting to be discovered. Interestingly, it seems a number of scientists had warned Einstein not to bother with gravity, the problem being too difficult. Thankfully Einstein ignored them and embarked upon a unification of special relativity with gravity, the result of which was nothing less than magic.

General relativity is governed by the *Einstein field equations*, which look like this:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

To the uninitiated, this is probably a rather terrifying collection of symbols. It is true the equations are very complicated but in essence they convey a beautifully simple message: matter or energy (T) distorts spacetime (g), and the resulting spacetime curvature (R) is what we experience as a gravitational field.

Imagine a rubber sheet (Figure 2). If we put something heavy into it like a bowling ball, the sheet will bend and distort. This is what happens to spacetime if we put something heavy into it like a star. The star distorts spacetime and this distortion in the sheet is what we experience as the force of gravity. In the above equation, the spacetime is represented by g, the bowling ball (or any matter) by T while R represents the curvature. Gravity, it would seem, is simply an effect of local spacetime geometry. This geometry can be changed by the presence of stars, planets and energy.

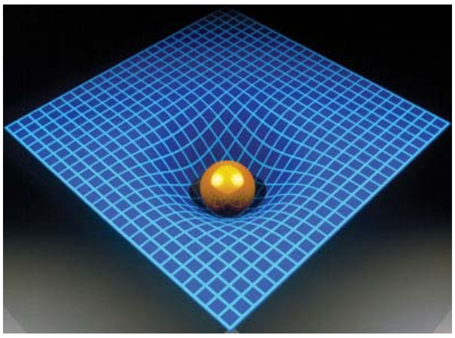

Figure 2. The rubber sheet analogy demonstrates the curvature of spacetime by putting a heavy object on a rubber sheet.

The general theory of relativity was published in 1915 and it was not widely accepted. It's possible that for many physicists, general relativity was just a step too far. In England, we have Arthur Eddington to thank for popularising it. He understood the significance of relativity, and he set out to test if it was right. General relativity was able to account for the perihelion precession of Mercury's orbit. It also made another curious prediction: photons could be influenced by gravity. If Einstein's theory was right, starlight could be bent by gravity. How on Earth could this be tested?

The answer came in the most spectacular fashion: a total eclipse of the Sun.

On 1919 May 29, there was to be a total eclipse of the Sun. Alas it was not visible from the UK, so Eddington set off on an expedition to Principie off the coast of west Africa. At the moment of totality, stars close to the eclipsed Sun should be visible and Eddington photographed them. When the plates were compared with a plate of the region six months earlier without the Sun present, it was seen that the starlight had indeed been bent; the gravitational field of the Sun had distorted the starlight by exactly the amount predicted by general relativity. Clearly the theory was on the right tracks and now had to be taken seriously.

Gravitational collapse

Solutions of the Einstein field equations are certain types of spacetime. The flat spacetime of special relativity (called Minkowski spacetime) is one example. There are however other, stranger solutions, including spacetimes where gravity is so strong that nothing could escape the pull of gravity – not even light. These are the black hole solutions.

The first such example was found mathematically by Karl Schwarzschild, quite remarkably, while he was serving in the German army during World War I. Schwarzschild found an object of extreme gravitational strength – at its heart was a singularity, a point of infinite density. The singularity was hidden from view by an *event horizon*, a surface surrounding the singularity. If anything crossed the event horizon it was doomed to fall into the singularity. Not even light could escape.

The object he had found is what we now call a *black hole*. Their discovery was met largely with indifference. Many physicists believed them to be a quirky solution to the equations of general relativity that were not physically real. After all, how could such an object come to exist?

Advances in the theory of stellar evolution would change that and provide some interesting insights into the formation of black holes. Stars, like people, are born, they live out their lives and then they die. How a star dies depends on its mass. Small stars between 0.7–8 solar masses end their lives in a quiet way. They become red giants and eventually shed their outer atmospheres to become that splendid object – a planetary nebula. Stars between 8–25 solar masses end their lives in a far more spectacular way – they explode as supernovae. When this happens a single exploding star can outshine the entire galaxy in which it existed.

It is the stars of 25 solar masses or more which have the strangest deaths. These stars are so heavy that gravity dominates. In Figure 3 I have sketched a spacetime diagram showing the collapse. The star collapses under its own weight, the collapse continues until all of the mass of the star is compressed into a single a point – a singularity. The singularity is hidden from the external Universe by the event horizon. The star has now become a black hole, and these were no longer quirky solutions of general relativity – they were the gravestones of massive stars.

Black Holes ain't so Black!

It was once the prevailing view that the black hole was the final endpoint of massive stars. Once the black hole had formed it was an eternal object, and no other physical processes could take place. However, it was soon realised that if black holes exist in our universe, they can't pick and choose which laws of physics they obey. They can't just obey the laws of gravity, they must also obey the rules of thermodynamics. Thermodynamics is the study of heat and its relation to energy and work.

Entropy is the measure of the amount of 'disorder' there is in a closed system. The second law of thermodynamics states that 'entropy increases'. Moreover if a body has entropy, then it must have heat associated with it and it must therefore radiate. Everything in the Universe obeys the laws of thermodynamics, and if black holes exist they must obey it too. Let's look at how the second law works, and for this purpose, my cat Mia is most helpful.

Let us consider Figure 4, starting at box 1. I have just poured out a beer, and Mia is sitting happily by the table. In box 2, Mia (for reasons known only to herself) scents the table leg, which wobbles the table causing the beer to fall off the table (box 3). Finally the beer falls onto the floor and the glass breaks (box 4). Anyone who lives with a cat will recognise this as a perfectly normal sequence of events.

This is an example of entropy increasing. At the start, the system was stable, then entropy increased steadily until the beer ended up on the floor, and the cat outside. It never happens that the beer is on the floor, it comes into the glass and the glass and beer re-assemble themselves on the table. Entropy always increases.

What does this mean for black holes? A black hole does have entropy – this is the event horizon. The event horizon never decreases and so if the black hole has a measure of entropy, it must have heat and so, it must radiate. The idea that black holes radiate was first suggested by Jacob Bekenstein, and Stephen Hawking derived the surface temperature of a black hole to be:

$$T_{BH} = kA / 4L^2$$

where k is Boltzmann's constant, A is the surface

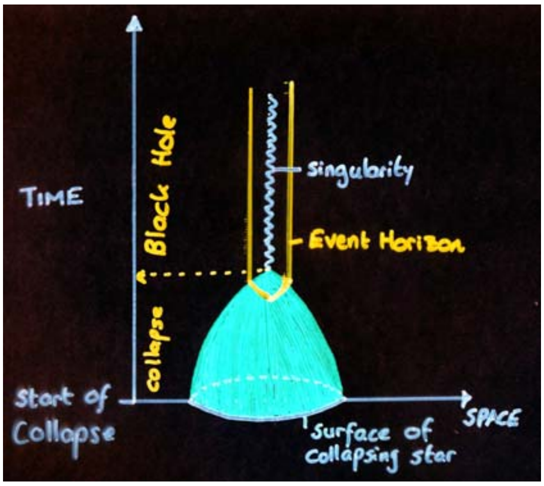

Figure 3. The gravitational collapse of a massive star.

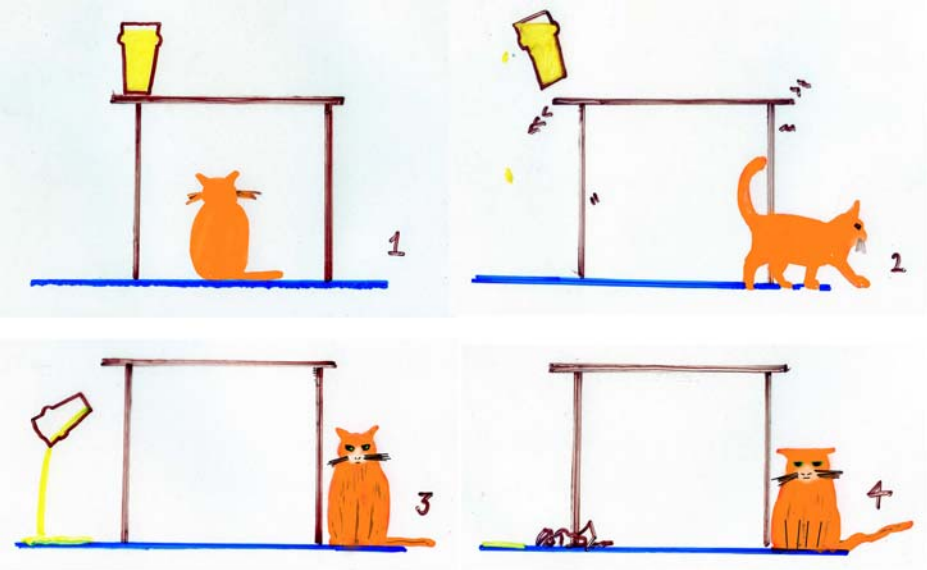

Figure 4. Mia, the cause of an increase in entropy which ends in a unforgivable loss of beer.

area of the black hole, and L is a constant containing Planck's constant. The interesting thing about the temperature of the black hole is that it is expressed in terms of fundamental constants, and does not in any way depend upon the details of the collapse that formed the singularity.

This result was remarkable; black holes were not static dead objects, they were dynamic. The radiation they emit consists of Hawking photons, and such radiation is small – it is only as the hole gets smaller that the radiation increases. Finally, we end up with a runaway process, and the black hole explodes, probably in a shower of gamma rays! The black hole is not eternal; over time it evaporates. This was probably one of the most remarkable results in black hole physics in the 20th century.

If black holes radiate – where does that radiation come from? The answer is found in quantum mechanics, in particular, the background vacuum energy of the Universe. The vacuum energy is the lowest energy state of the Universe. Even empty space is not empty – at the quantum level, small fluctuations in the vacuum energy cause particles and anti-particles to be created in pairs. In general the pairs form then annihilate each other. This can happen on the surface of a black hole: more specifically on the event horizon. Vacuum fluctuations cause particle/anti-particle pairs to form on the horizon. Most are annihilated, however occasionally one escapes and its opposite number falls into the black hole. This is the mechanism by which Hawking radiation occurs.

A future for theoretical physics?

As you can see, black holes have come a long way. They are now a staple of modern theoretical physics research. My own research interests at the University of Leicester are concerned with Hawking radiation. People are also looking into different types of black holes and their evolution. More than this, black holes may be the means of providing us with that much sought after prize: a quantum theory of gravity.

There is a slight problem with Physics at the moment. The two dominant theories in physics (which have successfully allowed the development of all sorts of technologies) are quantum mechanics and general relativity. General relativity has had many successes, indeed it has many down to earth uses for things like like GPS systems. Similarly quantum mechanics has allowed the development of the microchip, which is allowing me to bash out this paper on my computer to tell you why we should be interested in black holes.

However, general relativity and quantum mechanics fundamentally disagree on the nature of spacetime. GR tells us that spacetime is flexible, distorting in the presence of matter and energy, while QM tells us that spacetime is a sort of fixed stage, a rigid theatre on whose boards the elementary particles of nature dance to produce matter. How can it be, that we have two highly successful theories, and yet they disagree so radically on something so fundamental?

What is needed is a quantum theory of gravity: a theory which explains gravity at the macroscopic scale. A natural place to investigate how such a theory might look is the boundary of a black hole, since we have a very intense gravitational field and quantum mechanical effects happening near by.

What would such a theory look like? Personally I think it would have to be quite radical, perhaps disturbingly so. In the same way that Einstein's notions of matter, energy, space and time revolutionised physics a century or so ago, I believe a theory of quantum gravity will introduce fresh new ways of viewing nature. At the moment, the two main theories are string theory and loop quantum gravity. Both of these are highly complex, and while they entail some beautiful mathematics and original thought they have as yet to make any testable predictions, and to me, they just do not feel right.

Whatever a new theory has to say about gravity, I'm certain the black hole will have played its part in some way in its formation, though we might come to view such objects in a profoundly new way. We live in exciting times. With current particle accelerators we may soon be able to look for super-symmetry, which is essential for string theory. If we don't find it, that would be even more interesting. Perhaps it is time for another revolution.

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